Observations of non-Rayleigh statistics in the approach to photon localization

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We measure the distribution of intensity of microwave radiation transmitted through absorbing random waveguides of lengths L up to localization length ξ. For large intensity values the distribution is given by a negative stretched exponential to the 1/2 power, in agreement with predictions by Nieuwenhuizen and van Rossum [Phys. Rev. Lett. **74,** 2674 (1995)] for diffusing waves in nonabsorbing samples, as opposed to a negative exponential given by Rayleigh statistics. The intensity distribution is well described by a transform derived by Kogan and Kaveh [Phys. Rev. B **52,** R3813 (1995)] of the measured distribution of total transmission.

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The Rayleigh intensity distribution for multiply scattered light is the touchstone of statistical optics. It is derived under the assumption that the scattered field can be represented as a superposition of a large number of statistically independent partial waves. When the number of uncorrelated contributions to the field becomes large without limit, the probability distributions of the real and the imaginary components of the field for a single polarization component approach Gaussian distributions with zero average.[1] This gives a negative exponential distribution for the polarized intensity transmitted into mode b for an incident mode a, $P(s_{ab} \equiv T_{ab}/\langle T_{ab}\rangle) = \exp(-s_{ab})$, where $\langle \ldots \rangle$ denotes the ensemble average.[1–5] Under the same assumptions, the distributions of the total transmission, which is the sum over all output modes for a single input mode, $T_a = \sum_b T_{ab}$, and the transmittance, which is the sum over all output and input modes, $T = \sum_a T_a$, and is analogous to the electronic conductance, approach δ-functions. In actual experiments, however, the assumptions made in this model are violated to a greater or lesser extent. First, the number of transverse modes, N, is limited by the illuminated area A and the wavelength of radiation λ, $N = 2\pi A/\lambda^2$. Second, in addition to such short-range intensity correlation governed by correlation in the field that gives rise to the C_1 term[3,6,7] in the intensity correlation function, the long-range[8–12] and the infinite-range[9,13,14] intensity correlations, the C_2 and C_3 terms, respectively, lead to enhanced fluctuations in total transmission and conductance, respectively. In this Letter we concentrate on the intensity distribution and present measurements of the intensity distribution of microwave radiation in random waveguides with lengths up to the localization length ξ. As the sample length increases we find increasing deviations from Rayleigh statistics. For large values of s_{ab} the distribution, even at the localization threshold and in the presence of strong absorption, is well described by a negative stretched exponential to the power of 1/2, in agreement with recent calculations for diffusive propagation in nonabsorbing samples.[15,16] We also find that the measured intensity distribution is given by a transform of the measured distribution of total transmission, using a relationship derived by Kogan and Kaveh for nonabsorbing samples.[16]

The assumptions underlying Rayleigh statistics in nonabsorbing samples can be expressed by a single condition, $g \gg 1$, where g is the ensemble average of the transmittance. For diffusive waves $g = Nl/L$, where l is the transport mean free path. According to the scaling theory of localization,[17,18] the localization threshold is reached when $g = 1$, thus determining a localization length $\xi = Nl$. For nonabsorbing samples the statistics of wave transport can be described by use of the single parameter g. In the diffusive regime of propagation, to leading order in the small parameter $1/g$, the degree of intensity correlation between output modes b and b' for a single incident mode a, $\langle \delta s_{ab} \delta s_{ab'} \rangle$, where $\delta s_{ab} = s_{ab} - 1$, equals $2/3g$.[10,15,16,19] Deviations from Rayleigh statistics were observed in microwave experiments in samples with $g \approx 10$ (Refs. 5 and 20) and related to the degree of spatial intensity correlation.[20] First-order corrections to the Rayleigh distribution were calculated by Shnerb and Kaveh[21] and later by Kogan *et al.*, who found that the tail of the distribution was a negative stretched exponential to the power of 2/3 for $s_{ab} \gg g$.[22] More recently, Nieuwenhuizen and van Rossum[15] and Kogan and Kaveh[16] included higher-order corrections and found that the tail of the distribution is given by

$$P(s_{ab}) \sim \exp(-2\sqrt{g s_{ab}}\,). \qquad (1)$$

In the previous experiments in which a stretched exponential tail was observed,[5,20] however, the intensity distribution was measured only for $s_{ab} \lesssim g$, and the asymptotic decay of the distribution could not be ascertained.

Here we report measurements of intensity transmitted through random waveguides with transverse dimensions much less than L. For this quasi-one-dimensional geometry we expect that the modes will be completely mixed. The intensity distribution measured at any point in the output speckle pattern is therefore characteristic of transport in the sample as a whole. Statistics in this geometry can be calculated by use of random matrix theory.[23] Kogan and Kaveh

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used a random matrix theory approach to obtain a relationship between $P(s_{ab})$ and $P(s_a \equiv T_a/\langle T_a\rangle)$ in nonabsorbing quasi-one-dimensional samples in the limit $N \gg 1$.[16] They found that

$$P(s_{ab}) = \int_0^\infty \frac{ds_a}{s_a} P(s_a)\exp\left(-\frac{s_{ab}}{s_a}\right). \quad (2)$$

Equation (2) shows that negative exponential statistics obtain only when the distribution of total transmission is a δ-function.

We obtain the intensity distribution from measurements of the field transmitted through an ensemble of random configurations of 1.27-cm-diameter polystyrene spheres inside a copper tube. The waveguide geometry restricts transverse diffusion and, therefore, the number of modes N. Tubes with diameters $d = 5.0$, 7.5 cm and lengths from 50 to 520 cm are used. The samples have filling fractions of 0.52 and 0.55 for $d = 5.0$ cm and $d = 7.5$ cm, respectively. The field transmitted through the sample is measured from 16.8 to 17.8 GHz in steps of 0.625 MHz by use of a Hewlett-Packard 8722C network analyzer. The radiation is coupled into and out of the sample by 0.4-cm wire antennas placed 0.5 cm from the center of the output surface. To ensure that the distributions are not distorted by noise, we use an amplifier with an output power of 40 W for samples with lengths greater than 200 cm so that the average intensity is at least 300 times the noise. We rotate the sample tube between successive measurements to produce new configurations of scatterers. Dividing the intensity of each spectrum by the ensemble average intensity gives us s_{ab}. At least 2000 sample configurations are used in calculating the distribution for each sample length. A fit of the measured field–field correlation function with frequency shift to theory[24] gives an absorption length $L_a = 34 \pm 2$ cm for these samples. We note that in the presence of absorption g cannot serve as a useful measure of the proximity to the localization threshold.[25] In this case transport is characterized by the ratios L/ξ and L/L_a.[26] Using an effective medium index of refraction $n \approx 1.4$, we obtain $N \approx 90$ for the sample with $d = 5$ cm. In the frequency range of the measurements, $l = 5.5 \pm 0.5$ cm.[27] Thus we estimate that $\xi \approx 500$ cm for this sample and is approximately twice as large for $d = 7.5$ cm. Measurements of the total transmission distribution, $P(s_a)$, in absorbing samples showed that the full distribution can still be described by a single parameter g' obtained from the measured variance, $g' = 2/[3\,\mathrm{var}(s_a)]$,[25] which reduces to g in the absence of absorption. $P(s_{ab})$ is given by the transform of $P(s_a)$, then we would expect that the parameter g' is characteristic for the statistics of intensity as well.

In Fig. 1 we present an intensity spectrum for a single sample configuration for the sample with $L = 520$ cm and $d = 5$ cm, in which we observe fluctuations as large as $s_{ab} \approx 50$. Although strong absorption leads to dramatic reduction of the average transmission, large intensity fluctuations and, consequently, in broad distributions. The measured distributions for three samples are presented in Fig. 2. The distribution for sample (a), for which $L/\xi \approx 1/15$, is close to a negative exponential. As L/ξ approaches unity, however, the measured distributions exhibit increasing deviations from Rayleigh statistics. Since N is fixed, we associate these deviations with increasing degrees of nonlocal intensity correlation ($C_2 + C_3$) as L approaches ξ.

We investigate the tail of the intensity distribution in samples in which $P(s_{ab})$ is reliably measured for $s_{ab} \gg \xi/L$ which, in the absence of absorption, equals g. We fit the expression in relation (1) to the measurements, using g as a free parameter. The result of the fit for sample (b), which is made in the range $10 \le s_{ab} \le 20$, is presented as a dotted line in Fig. 3. Relation (1) is found to provide a good description of the tail of the distribution even at $L \approx \xi$, even though the theoretical expression was derived for diffusive transport in nonabsorbing samples. For the samples for which $P(s_{ab})$ is also measured,[25] the values of the fitting parameter obtained from the fit

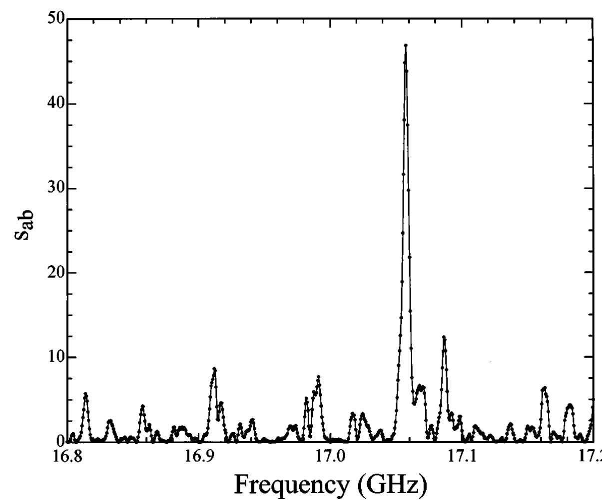

Fig. 1. Intensity spectrum for a single sample configuration for the sample with $L = 520$ cm ($\approx\xi$) and $d = 5$ cm.

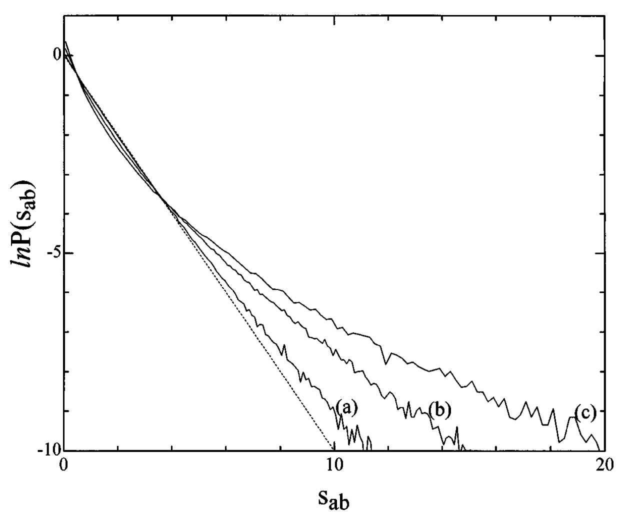

Fig. 2. Measured intensity distributions for three samples: (a) $L = 67$ cm, $d = 7.5$ cm, $L/\xi \approx 1/15$; (b) $L = 200$ cm, $d = 5$ cm, $L/\xi \approx 2/5$; (c) $L = 520$ cm, $d = 5$ cm, $L/\xi \approx 1$. The Rayleigh distribution is represented by the dotted line.

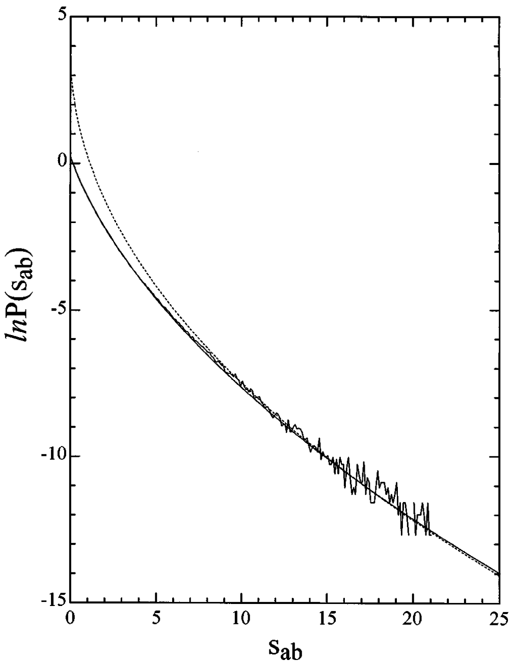

Fig. 3. Comparison of experiment and theory for sample (b) from Fig. 2. The smooth solid curve represents the transform of the total transmission distribution measured in Ref. 25. The dotted curve gives the fit to the tail with relation (1). The value of the fitting parameter of 3.20 ± 0.12 obtained from the tail of $P(s_{ab})$ is close to the value of $g' = 3.06 \pm 0.07$ obtained from total transmission measurements for the same sample.[25]

are within less than 10% of the values of the parameter g' found in Ref. 25. It appears that the degree of spatial intensity correlation $\langle \delta s_{ab} \delta s_{ab'} \rangle$ [=var(s_a)] that determines g' is the characteristic parameter for the intensity distribution as well, indicating that the effects of both absorption and localization on wave statistics are reflected in an essential way by this single parameter. For sample (c) the fit to the tail gives a value of 0.98 ± 0.03 for the fitting parameter in relation (1), which is characteristic for systems at the localization threshold.

We compare the measured intensity distributions with the transforms of the measured transmission distributions,[25] using Eq. (2) for sample lengths from 50 to 200 cm ($1.5 \lesssim L/L_a \lesssim 6$). The transform obtained for sample (b) is shown in Fig. 3. We note that for $s_{ab} \lesssim 5$ the measured and the calculated distributions essentially overlap. For all samples for which this comparison is made, we find good agreement between theory and experiment, which confirms the relationship between the distributions of intensity and total transmission [Eq. (2)] even in the presence of absorption.

In conclusion, we find that the microwave intensity distribution in random media broadens dramatically as the sample length increases, even in strongly absorbing samples. We find that the intensity distribution has a stretched exponential tail to the power of 1/2, in agreement with recent theoretical results. The measurements presented here confirm that $P(s_{ab})$ is given by a transform of $P(s_a)$, which unifies the study of the statistics of intensity and total transmission in random media. This work shows that the degree of spatial intensity correlation is a key parameter describing the distributions of intensity and total transmission in mesoscopic samples.

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