Photonic crystals built on contrast in attenuation

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We consider one-dimensional photonic crystals in which the contrast of layer impedances is due to differences in imaginary parts of layer dielectric permittivities. We show that even though in such structures there is no noticeable increase in the imaginary part of the Bloch wave number, there are frequency regions in which the transmission coefficient can rise or fall compared with a homogeneous slab of the same thickness and total absorption.

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The problem of electron motion in a periodic potential due to the crystalline lattice was first considered by Kronig and Penney[1] in 1931. In Ref. 1, the potential of a crystal was modeled by a one-dimensional periodic sequence of rectangular wells. This model explained the formation of band gaps in the electronic spectrum. While in quantum mechanics the Kronig–Penney model is purely illustrative, in optics it can be realized in one-dimensional photonic crystals (PCs).[2,3] The band gaps in optical spectra of PCs are the basis of all PC applications.[4]

Absorption causes smearing of the band gaps and leads to disappearance of the optical effects characteristic of PCs. Therefore, PCs are usually made of materials with minimal attenuation. One-dimensional PCs are the least sensitive to losses.[5] Such crystals consist of alternating layers with thicknesses comparable to the wavelength of the incident light. The main characteristic of such PCs is their contrast—the ratio of the real parts of the dielectric permittivities of the layers.

In this Brief Report, we consider PCs in which the impedance contrast is purely due to the differences between the imaginary parts of the permittivity of the layers. This model is essentially different from the Kronig–Penney model in solid state theory where one always deals with Hermitian operators.

In PCs, band gaps arise as a result of the resonant Bragg reflection; their widths are determined by the quality factor of these resonances and are proportional to the contrast of the dielectric permittivities (or to be precise, to their Fourier components). At the first sight, it seems that it is not critical whether the real or the imaginary parts of the permittivities form the layer contrast. However, in PCs where the contrast is achieved due to the differences between the imaginary parts only, according to the formal solution, the widths of the band gaps turn out to be also imaginary. The Bloch number for such a system has an imaginary part for all frequencies. Therefore, a periodic structure with a contrast of attenuations does not have distinct band gaps.[6] In this case, the role of losses is twofold. On one hand, losses cause scattering and the impedance contrast; therefore, the higher the loss is, the more pronounced should be effects caused by scattering. On the other hand, losses decrease the resonance Q factor and abate the effects of the resonance. It is not clear whether effects caused by wave scattering by layer boundaries can be observed at all. In this Brief Report, we investigate this problem.

We compare two types of structures: a finite thickness PC with contrast in attenuation and a homogeneous slab. The elementary cell of the PC consists of two layers with thicknesses d_1 and d_2 and dielectric constants ϵ_1 and ϵ_2, respectively. In order not to mix effects due to contrasts in the real and imaginary parts of the permittivity, we assume that $\mathrm{Re}(\epsilon_1)=\mathrm{Re}(\epsilon_2)$. Also for the sake of simplicity, we consider

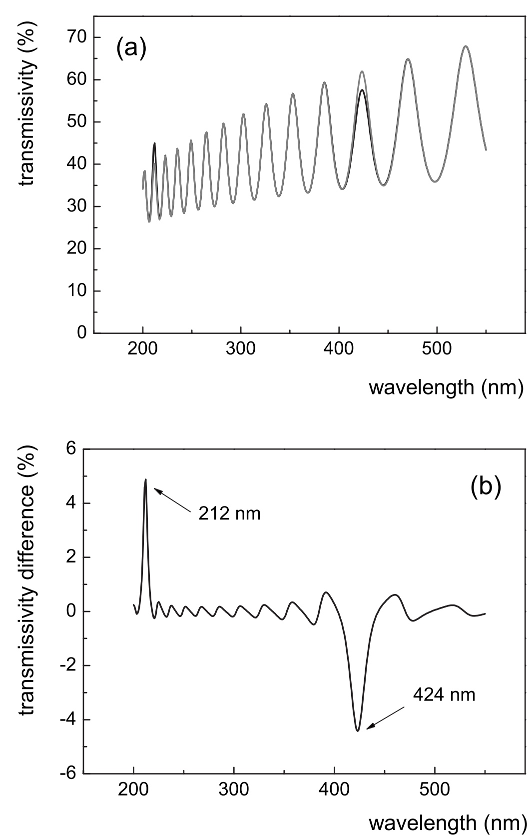

FIG. 1. (a) Transmission coefficients and (b) their difference of periodic (black line) and homogeneous (gray line) structures. The parameters of the systems are $\epsilon_1=7.0-0.0i$, $\epsilon_2=7.0-0.2i$, $d_1=50$ nm, and $d_2=30$ nm. The number of periods, N, of the PC is 10, and the total thickness of the PC and the homogeneous slab is $(d_1+d_2)N=800$ nm.

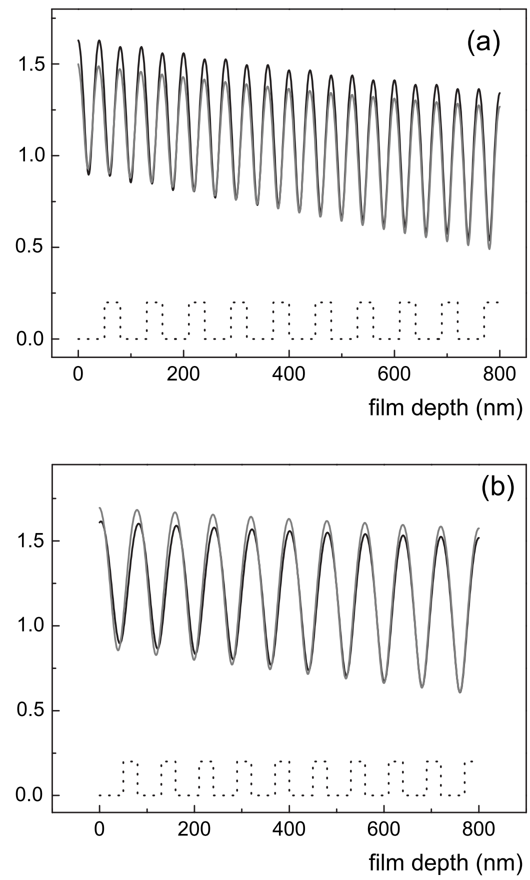

FIG. 2. The field distribution for the periodic (black line) and homogeneous (gray line) structures for the wavelengths of (a) 212 and (b) 414 nm.

$\mathrm{Im}(\epsilon_1)=0$ and $\mathrm{Im}(\epsilon_2)\neq 0$. The thicknesses of the PC sample and the slab are the same. The dielectric constant of the homogeneous slab, ϵ_{eff}, is chosen so that its properties are close to the averaged dielectric properties of the PC, $\epsilon_{\mathrm{eff}}=\langle\epsilon\rangle$. In our case this gives $\epsilon_{\mathrm{eff}}=\mathrm{Re}(\epsilon_1)+i\,\mathrm{Im}(\epsilon_2)d_2/(d_1+d_2)$. This approximation works well in the long-wave approximation (up to the first band gap).[4,7]

The Bloch wave numbers of the PC, k_B, can be determined from Rytov's formula,[5,7]

$$\cos(k_B d)=\cos(k_1 d_1)\cos(k_2 d_2)-\frac{k_1^2+k_2^2}{2\sqrt{k_1 k_2}}\sin(k_1 d_1)\sin(k_1 d_1), \tag{1}$$

where

$$d=d_1+d_2, \quad k_n=k\sqrt{\epsilon_n}, \quad k=\frac{\omega}{c}.$$

Using the transfer matrix method,[8] we calculated the transmission coefficients and the electric field distributions for a structure of 20 alternating layers with thicknesses and dielectric constants $\epsilon_1=7.0+0.0i$, $d_1=50$ nm and $\epsilon_2=7.0+0.2i$, $d_2=30$ nm. The results are shown in Figs. 1 and 2. The data shown in these figures were obtained for parameters corresponding to common optical materials used in the visible spectral region. Varying the parameters would result in numerical changes only, without altering the effects discussed below. As one can see in Fig. 1, the transmission coefficients of periodic and homogeneous structures differ by not more than 0.1% for the whole spectrum except in two narrow regions near 212 and 424 nm, where the difference exceeds 4% and the deviations have opposite signs. Thus, the transmission coefficients of the PC with the contrast in attenuation and the homogeneous slab of the same total thickness and total absorption oscillate with the change in the wavelength due to the interference of waves reflected from the outside boundaries. However, the PC is more transparent than the homogeneous slab at 212 nm and less transparent at 424 nm.

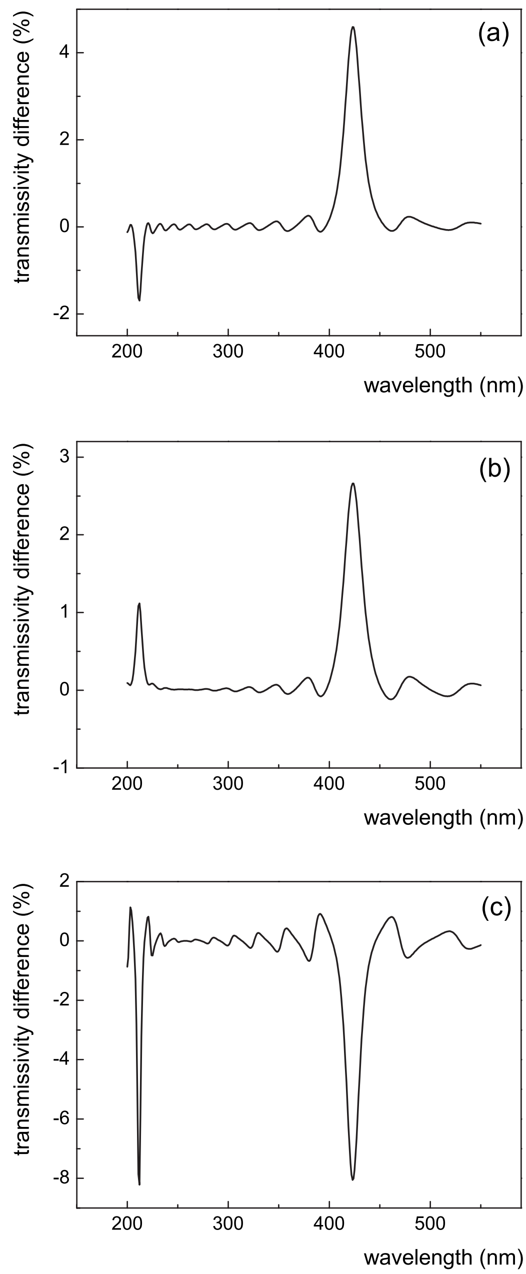

FIG. 3. Transmission coefficients difference for periodic and homogeneous structures with (a) $d_1=30$ nm and $d_2=50$ nm, (b) $d_1=10$ nm and $d_2=70$ nm, and (c) $d_1=70$ nm and $d_2=10$ nm.

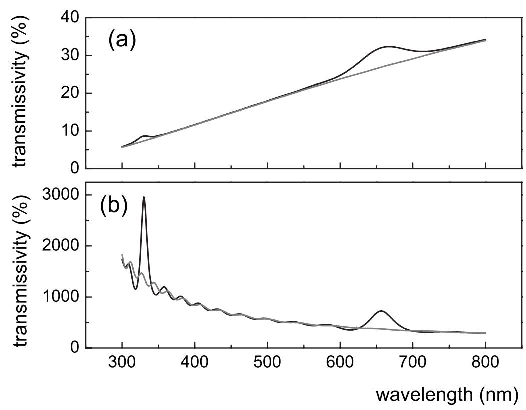

FIG. 4. The transmission coefficients of the periodic (line) and homogeneous (dots) systems. (a) Lossy systems with parameters $\epsilon_1=1.2-0.0i$, $\epsilon_2=1.2-0.15i$, $d_1=200$ nm, and $d_2=100$ nm. The number of periods of the PC is 10; the total thickness of the PC and the homogeneous slab is 3000 nm. (b) Systems with gain; all parameters are the same as in (a) except $\epsilon_2=1.2+0.15i$.

The wavelengths, at which transmissions through the PC and the homogeneous slab are different, are approximately equal to an integer number of the optical lengths of the PC period. For the first deviation, the wavelength is approximately equal to one period, $\sqrt{\epsilon}(d_1+d_2)=\sqrt{7}(50+30)=212$ nm, and an integer number of wavelengths fit into the length of the crystal. At the second region, 424 nm, two PC periods are equal to the wavelength. For both wavelengths, the real part of the cosine in Eq. (1) becomes greater than unity, which usually indicates the onset of a band gap.

In order to investigate the sign differences of deviations at 212 and 424 nm wavelengths, let us turn to the field distributions shown in Fig. 2. One can see that for the "resonance" at 212 nm, the field maxima for the periodic structure rise above those at the homogenous slab. The situation is reversed for the resonance at 424 nm. Higher (lower) fields lead to a higher (lower) transmission coefficient. These field differences occur because when an integer number of wavelengths fit the crystal length, the standing waves are formed along with the propagating waves. If the maxima of the standing wave locate at lossy layers, the transmission coefficient decreases and vice versa. Therefore, by changing d_1 and d_2 while keeping d_1+d_2 constant, one can achieve any combination of deviation signs (see Fig. 3).

These effects can be made more pronounced by increasing the loss contrast. For illustration, in Fig. 4(a) we show the transmission coefficient for a less realistic system with high loss. Even though the loss tangent for the system shown is high, the difference between transmission coefficients for periodic and homogeneous systems is near 6% at wavelengths of 330 and 658 nm. We can expect a dramatic increase in the transmission when lossy layers are replaced by layers with gain. Indeed, as one can see in Fig. 4(b), for a crystal with the same parameters as those in Fig. 4(a) but with the opposite sign of the imaginary part of the permittivity ϵ_2, at the wavelength of 330 nm the transmission coefficient of the PC is more than two times greater than that of the homogeneous slab.

To summarize, we show that in PCs built only on contrast of attenuations there are peculiarities of the transmission coefficient caused by the wave interference due to the periodically modulated impedance. This situation can be realized experimentally by consecutive sputtering of two materials with equal refraction indices or by creating periodic absorbing centers in a uniform material.

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