

independent. In the approximation considered, the width of the synchronization region is proportional to the field amplitude. Within the Arnold tongue, if the energy of the interaction of the NP with the external field is smaller than the QD-NP interaction, the amplitude of the auto-oscillations depends weakly upon the amplitude of the external field, which in this case plays the role of a synchronizer.

When $\Delta \rightarrow 0$ the threshold field value E_{synch} is determined by the equation $(\mu_{NP} E_{synch} / \hbar)^2 = (D_0 - D_{th}) \Delta^2 (\tau_a / 4\tau_D)$. For large values of the detuning, E_{synch} becomes independent of Δ (see Fig. 3). Taking into account that $\tau_a \sim 10^{-14} s^{-1}$ and $\tau_\sigma \sim 10^{-11} s^{-1}$ for plasmonic NPs, one can estimate the asymptotic value of the Arnold tongue boundary, E_{synch}^* . Our numerical calculations show that $E_{synch}(\Delta)$ tends to a plateau for $\Delta \sim 2 \cdot 10^{11} s^{-1}$, which gives $E_{synch}^* \sim 3 \cdot 10^3 V/m$. This agrees with calculations of [3,10] in which wave propagation in the system of spasers was considered. In [3,10] the amplitude of the incident wave was a few orders of magnitude larger than E_{synch}^* . Such a field is comparable with the near field inside the spaser. In this case, spaser becomes synchronized with the external field for any value of detuning; it ceases to be an autonomous system and responds linearly to the external field as can be seen in Fig. 3. For such strong fields, losses cannot be compensated exactly. Moreover, our numerical calculations show that for a very strong field, the population inversion of the QD decreases, the excitation of SPs by QDs is inhibited, and the spaser becomes just a lossy NP. Thus, when detuning is substantial, spasers cannot be used as active inclusions for the perfect lens because both loss and amplification destroy the perfect image, so that the exact compensation of loss is required.

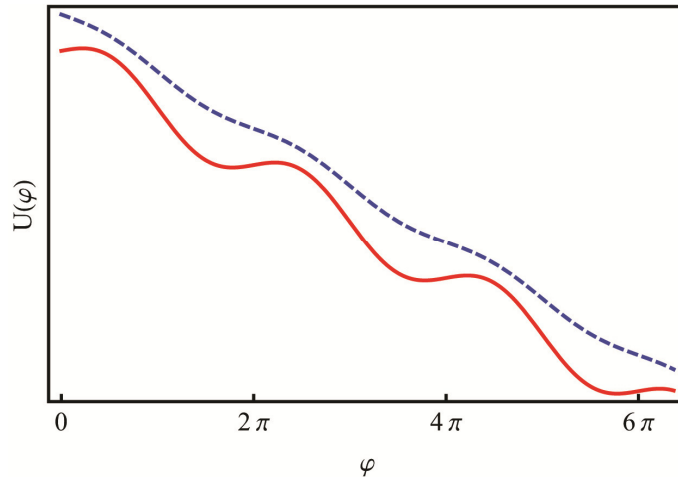


Fig. 4. The potential $U(\varphi)$ for $|\xi / \Delta| < 1$ (dashed blue line) and $|\xi / \Delta| > 1$ (solid red line).

For applications it is important to know not only a response of a single spaser to an external electromagnetic wave but also the behavior of a system of interacting spasers randomly or regularly distributed in the dielectric matrix. The interaction between spasers may lead to a cooperative behavior of spasers. This problem is a subject of our future research.

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