

Effects of inhomogeneous broadening on reflection spectra of Bragg multiple quantum well structures with a defect

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(Received 24 June 2003; revised manuscript received 3 September 2003; published 17 February 2004)

The reflection spectrum of a multiple quantum well structure with an inserted defect is considered. The defect is characterized by the exciton frequency ω_d different from that of the host's wells. Effects of both homogeneous γ and inhomogeneous broadenings Δ on a modification of the reflection spectrum caused by the defect are studied. In the case when the penetration length at the frequency ω_d is smaller than the length of the system, the modification of the spectrum is quite sharp and is noticeable up to very high values of Δ . Moreover, the position of the minimum of the reflection is shifted and this shift leads to a suppression of the effect of inhomogeneous broadening. This results in a significant drop of the reflection. A method of measuring γ and Δ from a single reflection spectrum is suggested.

DOI: 10.1103/PhysRevB.69.075308

PACS number(s): 78.67.De, 42.70.Qs, 71.36.+c, 42.25.Bs

I. INTRODUCTION

Optical properties of multiple quantum well (MQW) structures have been attracting a great deal of attention during the past decade.¹⁻¹⁶ The main motivation for this interest is the potential in these systems for effective control of the light-matter interaction. Quantum wells in MQW structures are separated from each other by relatively thick barriers, which prevent a direct interaction between excitons localized in different wells. They however, can be coupled by a radiative optical field, and their optical properties, therefore, become very sensitive to the arrangement of the structures. Most of the initial research in this area was focused on properties of periodic MQW's, which consist of identical quantum wells separated by identical barriers. The radiative coupling in this case gives rise to MQW polaritons—coherently coupled oscillations of the exciton polarization and light. In structures with a small number of wells, the spectrum of these collective excitations consists of a discrete number of quasistationary (radiative) modes with a redistributed oscillator strength: the modes can be classified as superadiant or subradiant.^{2,3,5} With an increase of the number of wells in the structure, the radiative lifetime of the latter decreases, and the lifetime of the former increases. When the number of periods in the structure becomes large enough, the modes of the MQW are more conveniently described in terms of stationary polaritons of infinite periodic structures.¹³ The spectrum of polaritons in this case consists of two branches separated by a polariton gap, which is normally proportional to the light-exciton coupling constant Γ_0 . There exists, however, a special arrangement of MQW's where the magnitude of the gap can be significantly increased. If the period of the MQW structure is made equal to the half-wavelength of the exciton radiation, the geometric, Bragg, resonance occurs at the same frequency as the exciton resonance. As a result, the band gap between two polariton branches becomes of the order of magnitude of $\sqrt{\Gamma_0\omega_0} \gg \Gamma_0$, where ω_0 is the frequency of the exciton transition. These, so called Bragg MQW structures were quite intensively studied both theoretically^{1,2,8,13} and experimentally.^{6,9-12} In the case of

structures with a small number of periods, the reconstruction of the optical spectra in Bragg MQW's can be described as a concentration of the oscillator strengths of all oscillators in one superradiant mode, while all other modes become dark.²

A presence of the large band gap in the spectrum of Bragg MQW polaritons invites attempts to introduce a defect into a structure in order to create local polariton states with frequencies in the band gap. This would affect the rate of spontaneous emission as well as other optical characteristics of the system. Such an opportunity was first considered in Ref. 14, and was studied in detail in Refs. 15 and 16. It was shown¹⁶ that by introducing different types of defects one can obtain optical spectra of a variety of shapes, which can be preengineered through the choice of the type of a defect, its position in the structure, the number of defects, etc. This fact makes these systems of interest for optoelectronic application. However, in order to be able to predict the optical spectra of realistic structures, any theoretical calculations must deal with the problem of inhomogeneous broadening, which is always present in these systems due to unavoidable structural disorder present in quantum wells. Calculations of Refs. 15 and 16 included inhomogeneous broadening using the linear dispersion theory.^{17,18} The linear dispersion theory treats inhomogeneous broadening as a simple addition to homogeneous broadening, ignoring therefore, effects of the motional narrowing, which have been very well studied in periodic multiple quantum wells.¹⁹⁻²¹ By doing so, the linear dispersion theory grossly overestimates the negative influence of inhomogeneous broadening, thereby giving unrealistically pessimistic predictions. The latter fact was verified in Ref. 22, where effects of the vertical disorder on the defect-induced features of the spectra were considered numerically.

More accurately inhomogeneous broadening can be studied with the help of the effective medium approximation, which was introduced in Refs. 23 and 24 on the basis of some qualitative arguments. In this approximation, a random susceptibility of a single quantum well is replaced by its value averaged over an ensemble of exciton frequencies, and therefore, one only needs to take into account the horizontal disorder, since on average all wells in the structure are as-

sumed identical. This approach was shown to agree well with experimental results for both cw and time-resolved spectra.²⁵ In this paper, we demonstrate explicitly the physical meaning of this approximation and the regions of its applicability.

The main objective of this paper is to study the effects of the inhomogeneous broadening in Bragg MQW structures with defects. Here we consider only one structure, namely a GaAlAs/GaAs structure with one of the wells replaced by a well with a different exciton frequency. A similar defect was considered in Ref. 16, where it was called an Ω defect; we retain this terminology in this paper. The consideration in Ref. 16 was focused mostly on the concept of local polariton states arising in infinite Bragg MQW structures, and on the effect of the resonant light tunneling arising due to these local states in long ideal Bragg MQW's. In this paper, we consider defect-induced modification of the optical spectra in more realistic systems. We take into consideration the inhomogeneous broadening and concentrate on structures with the number of periods readily available with current growth technologies. Keeping in mind the potential of these structures for applications, we study under which conditions the defect-induced resonant features of the spectra of realistic structures can be observed experimentally. We develop a general understanding of the spectral properties of Bragg MQW's in the presence of defects and disorder required for identifying structures most interesting from the experimental and application points of view. We also obtain a simplified analytical description for the main features of the spectra, valid in some limiting cases. This description is complemented by a detailed numerical analysis. One of the surprising results is that the features associated with the resonant tunneling via the local polariton state survive in the presence of disorder for much shorter lengths of the structure than was originally expected.^{15,16} This makes experimental observation and application of these effects significantly more attractive.

We also suggest a method of determining experimentally parameters of homogeneous and inhomogeneous broadening of a defect well in our structures from a single cw reflection spectrum. Such an opportunity seems to be quite exciting from the experimental point of view, since currently the separation of homogeneous and inhomogeneous broadenings requires the use of complicated time-resolved spectroscopic techniques. Attempts to develop a method for an independent extraction of the parameters of homogeneous and inhomogeneous broadenings from spectra of periodic MQW structures were undertaken in Refs. 23 and 26, but they also required either a time resolved spectroscopy or a not very reliable Fourier transformation of the original cw spectra.

II. REFLECTION SPECTRUM OF A MQW STRUCTURE

Within the framework of the linear nonlocal response theory, propagation of an electromagnetic wave in a multiple quantum well structure is governed by the Maxwell equations

$$\nabla \times (\nabla \times \mathbf{E}) = \frac{\omega^2}{c^2} (\epsilon_\infty \mathbf{E} + 4\pi \mathbf{P}_{ext}), \quad (1)$$

where ϵ_∞ is the background dielectric constant assumed to be the same along the structure, P_{ext} is the excitonic contribution to the polarization defined by

$$\mathbf{P}_{ext}(z) = \int \tilde{\chi}(\omega, z, z') \mathbf{E}(z') dz', \quad (2)$$

and the susceptibility $\tilde{\chi}$ is

$$\tilde{\chi}(\omega, z, z') = \tilde{\chi}(\omega) \Phi(z) \Phi(z'), \quad (3)$$

where $\Phi(z)$ is the exciton envelope function and z is the growth direction. Considering only the $1s$ heavy-hole exciton states and neglecting the in-plane dispersion of excitons, the susceptibility can be written as²⁷⁻²⁹

$$\tilde{\chi}(\omega) = \frac{\alpha}{\omega_0 - \omega - i\gamma}, \quad (4)$$

where ω_0 is the exciton resonance frequency, γ is the exciton relaxation rate due to inelastic processes $\alpha = \epsilon_\infty \omega_{LT} a_B^3 \omega_0^2 / 4c^2$, ω_{LT} is the exciton longitudinal-transverse splitting, and a_B is the bulk exciton Bohr radius.

The reflection spectrum of MQW structures is effectively described by the transfer-matrix method. For waves incident in the growth direction of the structure, the transfer matrix describing propagation of light across a single quantum well in the basis of incident and reflected waves is²

$$T = \frac{1}{t} \begin{pmatrix} t^2 - r^2 & r \\ -r & t \end{pmatrix}, \quad (5)$$

where r and t are the reflection and transmission coefficients of a single quantum well,

$$r = \frac{e^{i\phi} \eta}{1 - i\eta}, \quad t = \frac{e^{i\phi}}{1 - i\eta}, \quad (6)$$

$\phi = kd$, $k = \sqrt{\epsilon_\infty} \omega / c$ is the wave number of the electromagnetic wave, d is the period of the MQW structure (the sum of widths of the barrier and the quantum well),

$$\eta = \frac{\Gamma_0 \tilde{\chi}}{\alpha} = \frac{\Gamma_0}{\omega_0 - \omega - i\gamma}, \quad (7)$$

Γ_0 is the effective radiative rate

$$\Gamma_0 = \frac{2\pi\alpha k}{\epsilon_\infty} \left[\int dz \Phi(z) \cos kz \right]^2, \quad (8)$$

and we neglect the radiative shift of the exciton resonance frequency.

The parameter γ in the exciton susceptibility, Eq. (4), introduces the nonradiative homogeneous broadening due to inelastic dephasing of excitons. Inhomogeneous broadening results from fluctuations of the exciton transition frequency ω_0 in the plane of a well caused by, for example, imperfections of the interface between the well and the barrier layers and/or presence of impurities.

We are interested here only in the characteristics of waves reflected in the specular direction. As it is shown in the Appendix this component of the scattered field obeys Eq. (1)

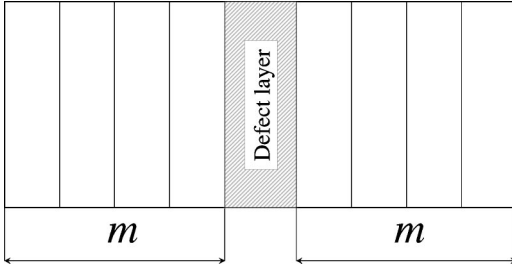


FIG. 1. A MQW structure with a defect.

with the susceptibility averaged with respect to the distribution function of the exciton frequencies $f(\omega_0)$

$$\chi = \int d\omega_0 f(\omega_0) \frac{\alpha}{\omega_0 - \omega - i\gamma}. \quad (9)$$

The inhomogeneous broadening is characterized, in this approach, by the variance of $f(\omega_0)$. This constitutes the effective media approximation.

The function χ replaces $\tilde{\chi}$ in Eq. (7) and determines a single-well transfer matrix of a broadened well,²³ which is then used to construct the transfer matrix describing the propagation of light through the entire Bragg MQW structure. The structure considered in this paper, a Bragg MQW with an Ω defect, consists of $N=2m+1$ quantum well-barrier layers which are all identical except for one, at the center, where the quantum well has a different frequency of the exciton resonance (Fig. 1). Such a defect can be produced either by changing the concentration of Al in the barriers surrounding the central well,^{30,31} or the width of the well itself³² during growth. In principle, both these changes will also affect the optical width of the defect layers, either because of the change in the background dielectric constant, or due to the change of the geometrical thickness of the well. However, in both cases this effect is negligible, and we deal here with the case of a pure Ω defect.

The total transfer matrix through the MQW structure consists of the product

$$M = T_h \dots T_h T_d T_h \dots T_h, \quad (10)$$

where T_h and T_d are the transfer matrices through the host and defect layers, respectively, described by the reflection and transmission coefficients $r_{h,d}$ and $t_{h,d}$. Substitution of the explicit expressions for T_h and T_d leads to a compact expression for the total transfer matrix in the basis of eigenvectors of the host transfer matrix T_h

$$M = \begin{pmatrix} e^{-\Lambda} M_- & a_+ A \\ -a_- A & e^{\Lambda} M_+ \end{pmatrix}, \quad (11)$$

where $\Lambda = N\lambda_h$,

$$M_{\pm} = e^{\pm(\lambda_d - \lambda_h)} \pm \frac{2e^{\mp\lambda_h}}{\sinh \lambda_h} \sinh^2 \frac{1}{2}(\lambda_d - \lambda_h), \quad (12)$$

and

$$A = \frac{\sin \phi}{\sinh \lambda_h} (\eta_d - \eta_h). \quad (13)$$

Here we introduced a_{\pm} , nonunit components of the eigenvectors of T_h ,

$$a_{\pm} = \frac{1 - e^{\pm\lambda_h} t_h}{r_h}, \quad (14)$$

and $\lambda_{h,d}$ are the eigenvalues of the host and defect quantum well's transfer matrices obeying the dispersion law in a periodic quantum well superlattice:¹⁻³

$$\cosh \lambda_{h,d} = \frac{1}{2} \text{Tr} T_{h,d} = \cos \phi - \eta_{h,d} \sin \phi. \quad (15)$$

In the case of an ideal system without homogeneous or inhomogeneous broadenings, this equation describes the band structure of the electromagnetic spectrum of MQW's, consisting of a number of bands separated by forbidden band gaps, defined as frequency regions where $\text{Re} \lambda_h \neq 0$. This real part describes the exponential decrease of the amplitude of an incident wave, and its inverse gives the value of the respective penetration length. In the allowed bands, λ_h is purely imaginary, and its imaginary part is the Bloch wave vector of the respective excitation. There are two types of the excitations here. In the vicinity of the exciton frequency ω_h , there exist two polariton branches separated by a polariton band gap.¹³ There are also pure photonic bands with the band boundaries at the geometrical resonances, $\phi(\omega_r) = n\pi$ ($n = 1, 2, \dots$). The size of the polariton gap strongly depends on the relation between ω_h and ω_r , and reaches the maximum value when they coincide, i.e. when $\phi(\omega_h) = \pi$ (the Bragg structure). For these frequencies, it is convenient to extract the imaginary part from λ_h and to present it in the form

$$\lambda_h = \kappa_h + i\pi. \quad (16)$$

If the coupling parameter Γ_0 is small, κ_h in the gap can be approximated by the following expression, which is obtained by expanding Eq. (15) near the resonance frequency

$$\kappa_h = \sqrt{\pi q (-2\eta_h - \pi q)}, \quad (17)$$

where q is the detuning from the Bragg resonance

$$q = \frac{\omega - \omega_h}{\omega_h}. \quad (18)$$

Taking into account the form of the susceptibility in an ideal system, we obtain

$$\kappa_h = \pi \sqrt{Q_G^2 - q^2}, \quad (19)$$

where

$$Q_G = \sqrt{\frac{2\Gamma_0}{\pi\omega_h}} \quad (20)$$

determines the boundary of the forbidden gap¹³ as a point where κ_h vanishes. The frequency, which corresponds to this

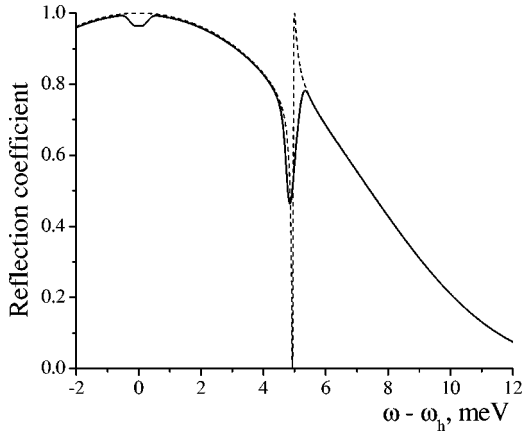


FIG. 2. A typical dependence of the reflection coefficient of a MQW structure with an embedded defect well in a neighborhood of the exciton frequency of the defect well. The dotted line shows the reflection for a lossless system, and the solid line corresponds to a broadened system (parameters are taken for GaAs/AlGaAs).

boundary determines the width of the gap as $\sqrt{2\Gamma_0\omega_h}/\pi$. This value is significantly greater than the respective width in the off-Bragg case, which is proportional to $\Gamma_0 \ll \omega_h$. In the presence of homogeneous and inhomogeneous broadenings, the notion of the band gap becomes ill defined, because κ_h does not vanish anywhere. At the gap boundary, for instance, it becomes complex valued

$$\kappa_h(\omega_G) = (1+i)\pi \sqrt{\frac{\gamma Q_G}{2\omega_h}}. \quad (21)$$

Nevertheless, if $\gamma \ll \sqrt{\Gamma_0\omega_h}$, which is the case in realistic systems, the optical spectra retain most of their properties specific for the gap region, and this concept provides a useful physical framework for discussing optical properties of MQW structures.

Once the transfer matrix M for the entire MQW structure is known the reflection and transmission coefficients can be expressed in terms of its elements m_{ij} as

$$r_{MQW} = \frac{-m_{11} + m_{12} - m_{21} + m_{22}}{a_-(m_{12} + m_{22}) - a_+(m_{11} + m_{21})},$$

$$t_{MQW} = \frac{a_- - a_+}{a_-(m_{12} + m_{22}) - a_+(m_{11} + m_{21})}. \quad (22)$$

A defect inserted into the structure leads to a modification of the reflection spectrum of the MQW in the vicinity of the exciton frequency ω_d of the defect well. We are interested in the situation when ω_d lies within the polariton band gap of the ideal host structure, because in this case the defect produces the most prominent changes in the spectra. A typical form of such a modification in broadened systems is shown in Fig. 2, and is characterized by the presence of a closely positioned minimum and maximum. In an ideal system, this form of reflection would correspond to a Fano-like resonance in the transmission with the latter swinging from zero to unity over a narrow frequency interval.¹⁶ In the presence of absorption and inhomogeneous broadening, the resonance

behavior of the transmission is smeared out, while the resonance in reflection, as we can see, survives.

In order to analyze the form of the reflection spectra, we represent the reflection coefficient in the form

$$r_{MQW} = \frac{r_0}{1 - r_{add}}, \quad (23)$$

where

$$r_0 = \frac{2\sinh(\Lambda)}{a_-e^\Lambda - a_+e^{-\Lambda}} = -\frac{\eta_h}{\alpha + i\coth(\Lambda)\sinh\lambda_h} \quad (24)$$

is the reflection coefficient of a pure MQW structure (without a defect) with the length N , $\alpha = \sin\phi + \eta_h\cos\phi$, and r_{add} introduces the modification of the reflection caused by the defect

$$r_{add} = (\eta_d - \eta_h)\sin\phi \frac{\sinh\lambda_h + i\eta_h\sinh\Lambda}{\eta_h\cosh\Lambda - \alpha} \times \frac{1}{A(\alpha + \eta_h\cosh\Lambda) - \eta_h\sinh\Lambda}. \quad (25)$$

This expression allows for making some general conclusions regarding the effects of the defect and broadenings on the reflection spectrum. First of all, it should be noted that regardless of the value of the defect frequency ω_d , r_{add} vanishes at frequencies $\omega = \omega_h$ because of the phase factor $\sin\phi \approx -\pi q$.

Thus, in order to achieve a significant modification of the spectrum, it is necessary to choose ω_d as far away from ω_h as possible. In this case, however, we can immediately conclude that the broadening of the host wells does not significantly affect the defect-induced features of the reflection spectrum.

Indeed, the broadenings enter into Eq. (25) through the susceptibility η_h defined by Eq. (9). Let us rewrite this definition in the form

$$\eta_h = \int d\nu f(\nu) \frac{\tilde{\Gamma}}{\nu - q - i\tilde{\gamma}}, \quad (26)$$

where we have introduced $\tilde{\Gamma} = \Gamma_0/\omega_h$, $\tilde{\gamma} = \gamma/\omega_h$, $\nu = (\omega - \omega_h)/\omega_h$, and ω_h is the mean value of the exciton frequency. If the function $f(\nu)$ falls off with increasing ν fast enough, so that all its moments exist, we can approximate the integral for the frequencies $q/\tilde{\Delta}, q/\tilde{\gamma} \gg 1$ as

$$\eta_h \approx \tilde{\eta}_h \int d\nu f(\nu) \left(1 + \frac{\nu}{q + i\tilde{\gamma}} + \dots \right), \quad (27)$$

where

$$\tilde{\eta}_h = \frac{\Gamma_0}{\omega_h - \omega - i\gamma} \quad (28)$$

is the susceptibility in the absence of the inhomogeneous broadening. Noting that now integration of each term in the parentheses gives an appropriate central moment of ν we obtain

$$\eta_h \approx \tilde{\eta}_h \left(1 + \frac{\tilde{\Delta}^2}{q^2} + \dots \right). \quad (29)$$

Therefore, the corrections due to the inhomogeneous broadenings become small for frequencies, which are farther away from the central frequency than the inhomogeneous width Δ . Since the width of the polariton gap in Bragg MQW structures is significantly greater than the typical value of the inhomogeneous width, we can choose such a position of the defect frequency ω_d which is far enough from ω_h , and at the same time remains within the gap frequency region.

Significant diminishing of the effects due to disorder in optical spectra of periodic MQW for frequencies away from the resonance exciton frequency was obtained theoretically in Ref. 23 and observed experimentally in Refs. 33 and 34. This can also be seen in Fig. 2, where deviations from the spectra of an ideal structure (dashed line) caused by disorder in the host wells are significant only in the vicinity of ω_0 . The modification of the defect-induced features of the spectra, which takes place in the vicinity of ω_d are caused by broadenings of the excitons in the defect layer, and this is the only broadening, which has to be taken into consideration in this situation.

III. DEEP AND SHALLOW DEFECTS

While Eq. (25) is suitable for a general discussion and numerical calculations, it is too cumbersome for a detailed analytical analysis. For such purposes we consider the reflection coefficient for two limiting cases when the expression for r_{add} can be significantly simplified. The first limit corresponds to the situation when the penetration length of the electromagnetic wave at the frequency of the defect is much smaller than the length of the structure (we call it a deep defect), and the second is realized in the opposite case, when the system is much shorter than the penetration length (shallow defect).

A. Deep defect

Such situation was analyzed in Ref. 16 for the case of a lossless system. To determine the range of validity of these results in the case of broadened systems we represent the broadening as a sum of the homogeneous and inhomogeneous ones as in linear dispersion theory. The estimation of the effect of the broadenings on the reflection spectrum shows that when

$$\gamma + \Delta \lesssim 8\pi\omega_h \sqrt{Q_G^2 - \delta^2} \frac{e^{-2\Lambda} \delta^3}{Q_G^2}, \quad (30)$$

where $\delta = (\omega_d - \omega_h)/\omega_h$, the effect of the broadenings can be neglected, and the results of Ref. 16 are valid. This inequality, however, becomes broken for long systems, and the broadenings must be taken into account. In this case the ex-

ponentially small nonresonant terms in Eq. (25) can be neglected and the reflection in the vicinity of ω_d can be presented in the form

$$r = r_0 \frac{\Omega_d - \Gamma_0 D_d}{\Omega_d - \Gamma_0 D_d - 2ie^{-\Lambda} \delta^2 \omega_h}, \quad (31)$$

where $D_{d,h} = 1/\eta_{d,h}$ and

$$r_0 = \frac{1}{1 + D_h[\pi q + i\kappa_h(1 + 2e^{-2\Lambda})]} \quad (32)$$

is the approximation for the reflection coefficient of the structure without the defect for frequencies deeply inside the forbidden gap. We keep the term $\exp(-2\Lambda)$ in this expression in order to preserve the correct dependence of the reflection coefficient of the pure structure on its length. The frequency Ω_d ,

$$\Omega_d = \pi\Gamma_0 \frac{\delta}{\kappa_h}, \quad (33)$$

describes the shift of the position of the reflection resonance from the initial defect frequency ω_d . This shift is an important property of our structure, which takes place in both ideal and broadened systems.

Deriving Eq. (31) we also assumed that the exciton frequency of the defect well lies far enough from the frequency of the host wells, neglected the contribution of the host susceptibility η_h into the terms proportional to $\eta_d - \eta_h$, and dropped a frequency dependence of the nonresonant terms.

Equation (31) shows that when the defect well exciton frequency lies deeply inside the forbidden gap the effect of the defect on the reflection spectrum of the system exponentially decreases when the length of the MQW structure increases. This behavior is strikingly different from that of the respective ideal systems, where resonant tunneling results in the transmission equal to unity at the resonance regardless of the length of the system. One can see that homogeneous broadening severely suppresses this effect, as was anticipated in Ref. 16.

If the shift Ω_d of the resonance frequency from ω_d is large enough, so that ω_r is well separated from ω_d , the effects of the inhomogeneous broadening can be neglected. In this case, we can derive a simple approximate expression for the reflection coefficient in the vicinity of ω_r . The condition $\Omega_d \gg \Delta$ can, in principle, be fulfilled because $\kappa_h(\omega_d)$ decreases when the frequency goes to the edge of the stop band where κ_h is determined by Eq. (21) and for GaAs/AlGaAs MQW structures with $\omega_h = 1.49$ eV, $\Gamma_0 = 67$ μ eV, $\gamma = 12.6$ μ eV, and $\Delta = 290$ μ eV we obtain $\text{Re}[\Omega_d(\omega_h + \omega_G)]/\Delta \approx 6.3$.

In this case, in the vicinity of the resonance frequency we can approximate the susceptibility η_d by

$$\eta_d = \frac{\Gamma_0}{\omega_d - \omega - i\gamma} \quad (34)$$

and obtain that the resonance has a form of the Lorentz-type dip on the dependence of the reflection spectrum positioned at $q = \Omega_d$ with the depth H and the width W defined by expressions

$$H = |r_0|^2 \frac{1 + \gamma e^\Lambda / \omega_h \delta^2}{(1 + \gamma e^\Lambda / 2\omega_h \delta^2)^2}, \quad (35)$$

$$W = \gamma + e^{-\Lambda} \delta^2 \omega_h. \quad (36)$$

It should be noted, however, that while formally this approximation is valid even when ω_d is close to the edge of the forbidden gap, the deep defect approximation requires that $\Lambda \gg 1$. For GaAs/AlGaAs structures this means that $N > 1/\text{Re}(\kappa_h) \sim 2000$. The structures of this length are beyond current technological capabilities, so this case presents mostly theoretical interest.

In the opposite situation, when the frequency shift is small ($\Omega_d < \Delta$), i.e., when ω_d is not too close to the edge of the gap, the inhomogeneous broadening becomes important, and in order to estimate its contribution we use a Gaussian distribution of the exciton resonance frequencies in Eq. (9):

$$\eta_d = \frac{\Gamma_0}{\Delta \sqrt{\pi}} \int_{-\infty}^{\infty} d\omega_0 \frac{e^{-(\omega_0 - \omega_d)^2 / \Delta^2}}{\omega_0 - \omega - i\gamma}. \quad (37)$$

Using the function $w(\mu) = e^{-\mu^2} \text{erfc}(-i\mu)$, the integral can be written as

$$\eta = i\Gamma_0 w(\mu) \sqrt{\pi} / \Delta, \quad (38)$$

where $\mu = (\omega - \omega_d + i\gamma) / \Delta$. The small μ expansion³⁵

$$w(\mu) \approx 1 + 2i\mu / \sqrt{\pi}$$

allows us to obtain

$$D_d = \frac{2}{\Gamma_0 \pi} (\omega_d - \omega - i\tilde{\gamma}), \quad (39)$$

where $\tilde{\gamma}$ is the effective broadening,

$$\tilde{\gamma} = \gamma + \frac{\sqrt{\pi}}{2} \Delta. \quad (40)$$

One can see that in this case the inhomogeneous and homogeneous broadening combine to form a single broadening parameter $\tilde{\gamma}$, as it is assumed in the linear dispersion theory. The resonance on the reflection curve also has, in this case, a Lorentz-type dip centered at

$$q = \frac{\pi \Gamma_0 \delta}{2\omega_h \sqrt{\omega_G^2 - \delta^2}}, \quad (41)$$

with the depth and the width, respectively, equal to

$$H = |r_0|^2 \frac{2\pi e^{-\Lambda} \delta^2 \omega_h}{\tilde{\gamma}}, \quad W = \tilde{\gamma} + \pi e^{-\Lambda} \delta^2 \omega_h. \quad (42)$$

Because of the inhomogeneous broadening contribution, the effective parameter $\tilde{\gamma}$ becomes so large that $\pi e^{-\Lambda} \delta^2 \ll \tilde{\gamma} / \omega_h$, and the defect-induced reflection resonance becomes rather weak compared to the case considered previously.

B. Shallow defect

From the experimental point of view, a more attractive situation arises when the length of the structure is smaller than the penetration length. This situation, which can be called the case of a shallow defect, can be described within the same approximations as used in the preceding section, with an obvious exception of the treatment of terms proportional to e^Λ . Here we can expand the exponential function in terms of the powers of Λ . Finally we arrive at the following expression, which describes the defect-induced modification of the reflection spectra:

$$r = \frac{i\Gamma}{\omega_h - \omega + i(\gamma + \bar{\Gamma})} \frac{\Omega_s - \Gamma_0 D_d}{i\Gamma_0 - \Gamma_0 D_d}, \quad (43)$$

where $\bar{\Gamma}$ is the radiative width of the pure Bragg MQW structure, which is N -fold enhanced because of the formation of a superradiant mode⁴

$$\bar{\Gamma} = \frac{\Gamma_0 N}{1 - i\pi q N}. \quad (44)$$

This expression coincides with the results of Ref. 4 with the exception of the term proportional to qN , which was neglected in the previous papers. We keep this term to be able to consider the case when the detuning from the resonance point ω_h is not very small.³⁶

The distinctive feature of the shallow defect is that the reflection resonance does not have a Lorentz-like shape, which is typical for the deep defect when the resonant tunneling is suppressed. Instead, we have a reflection spectrum with a minimum at ω_- and a maximum at ω_+ . This is a surprising result, because the Fano-like behavior associated with the resonant tunneling is restored even though the length of the system is too short for effective tunneling to take place. It is convenient to describe the positions of these frequencies relative to the modified defect frequency,

$$\tilde{\omega}_d = \omega_d - \Omega_s, \quad (45)$$

where Ω_s is defined as

$$\Omega_s = \frac{\omega_d - \omega_h}{N}. \quad (46)$$

The positions of the extrema are shifted from $\tilde{\omega}_d$: ω_- toward the center of the gap, and ω_+ in the opposite direction. As a result, ω_- is well separated from ω_d , while ω_+ always lies in the close vicinity of the defect frequency.

For *short* systems, Ω_s can become larger than Δ ; for example, in GaAs/AlGaAs MQW structures, the condition $\Omega_s \gg \Delta$ is fulfilled when $N \leq N_0 = 10$. In this case, an approximate analytical description of the spectrum is possible again.

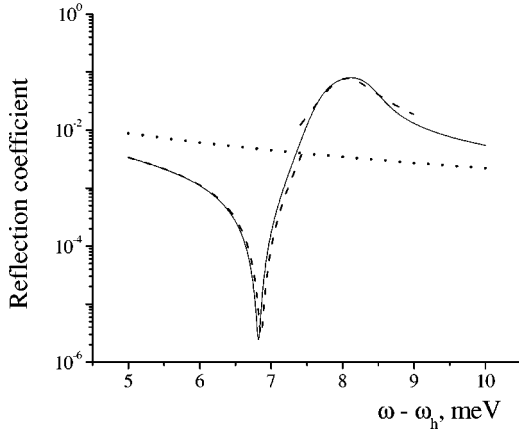


FIG. 3. Reflection coefficient near the exciton frequency of the shallow defect (solid line) for $N=7$. The dashed lines depict approximation using different expressions for the defect quantum well susceptibility at the vicinities of the extrema: near the minimum the inhomogeneous broadening is neglected, while in the vicinity of the maximum it is accounted for as a renormalization of homogeneous broadening [Eq. (40)]. For reference, the reflection coefficient of a pure MQW structure without a defect is shown (dotted line).

However, since the maximum and the minimum of the spectra lie at significantly different distances from ω_d , the description of these two spectral regions would require different approximations. The maximum of the reflectivity takes place close to the defect frequency, and therefore the inhomogeneous broadening near the maximum has to be taken into account. At the same time, $\omega_- - \omega_d \gg \Delta$, and the inhomogeneous broadening in this frequency region can be neglected. Thus, we can approximate D_d by Eq. (34) in the vicinity of ω_- and by Eq. (39) near ω_+ . Using these approximations we find that the minimum and the maximum of the reflection coefficient are at the frequencies

$$\omega_- = \omega_d - \Omega_s - \frac{\gamma^2}{\Omega_s}, \quad (47)$$

and

$$\omega_+ = \omega_d + \frac{1}{\pi}(\tilde{\Omega}_s - \Omega_s) + \frac{\Gamma_0 \tilde{\gamma}}{\tilde{\Omega}_s \Omega_s}(\tilde{\Omega}_s + \Omega_s) \quad (48)$$

respectively, where $\tilde{\Omega}_s = \sqrt{\Omega_s^2 + 4\tilde{\gamma}^2}$ and $\tilde{\gamma}$ is the effective broadening defined by Eq. (40). The values of the reflection at these points are

$$R_{min} = \frac{|\Gamma|^2 \gamma^2 N^4}{(\omega_d - \omega_h)^4 (N-1)^2},$$

$$R_{max} = \frac{|\Gamma|^2 (\tilde{\Omega}_s + \Omega_s)^2}{(\omega_+ - \omega_h)^2 (2\Gamma_0 + \pi \tilde{\gamma})^2}. \quad (49)$$

The exact and approximate forms of the reflectivity are compared in Fig. 3. One can see that these approximations give a satisfactory description of the reflectivity in the vicinities of the extrema for short systems.

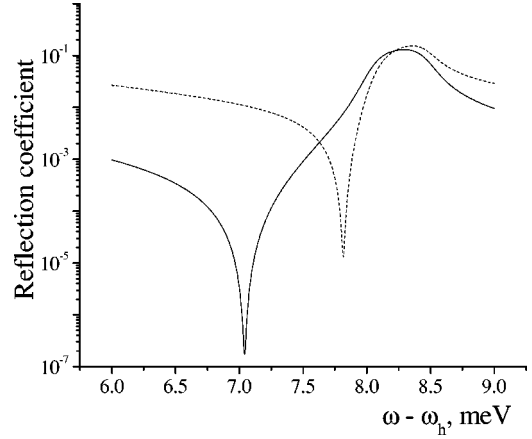


FIG. 4. Dependence of the reflection coefficient on the frequency in the neighborhood of the exciton frequency of the defect well for different lengths of the MQW structure (solid line $N=3$, dashed line $N=5$). The great difference of minimal reflections results from the shift of the resonance frequency Ω_s , Eq. (46).

The minimal value of the reflection is determined only by the small parameter of the homogeneous broadening γ and can therefore become very small. This fact reflects the suppression of the inhomogeneous broadening in this situation. When the length of the system increases, R_{min} grows as N^4 , however, when $N > N_0$, the inhomogeneous broadening starts coming into play: γ must be replaced with a larger effective broadening containing a contribution from Δ . This also leads to a significant increase in R_{min} . This behavior is illustrated in Fig. 4, where a comparison of the reflection coefficients for two MQW structures with different lengths is provided.

The maximum value of the reflection coefficient in this approximation depends very weakly on the number of wells in the system, and is of the order of the magnitude of the reflection from a single standing defect well in the exact resonance. This result means that for a small number of wells, ω_+ lies in the spectral region, where the host system is already almost transparent, and the presence of the host has only a small effect on the reflection properties of the system with the defect. At the same time, we would like to emphasize again that the minimum of the reflection is the result of the radiative coupling between the wells even in this case of short systems.

C. Characterization of the reflection spectra in the case of intermediate lengths

In the previous sections we examined special situations when the defect can be considered as either deep or shallow. Here we consider systems with intermediate lengths, when N is larger than N_0 , but is still smaller or of the order of magnitude of the penetration length. From the practical point of view, this case is of the greatest interest, since this interval of lengths is still easily accessible experimentally, and at the same time, it is expected that for such structures the defect-induced modifications of the spectrum become most prominent. Unfortunately, neither approximation used in the previ-

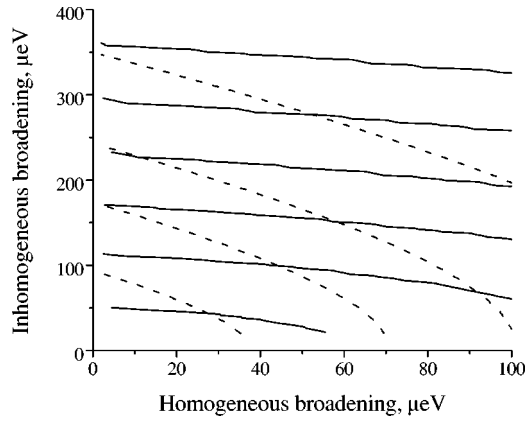


FIG. 5. Intersections of lines of constant height (dashed lines) and width (solid lines) of the resonance allow determination of the values of the homogeneous and inhomogeneous broadenings.

ous sections can be applied here, and we have to resort to a numerical treatment. Nevertheless, the qualitative understanding gained as a result of the previous analytical considerations, serves as a useful guide in analyzing and interpreting the numerical data.

As it was pointed out in the preceding section, when N becomes larger than N_0 , the position of the minimum of the reflection, ω_- moves closer to ω_d , and the inhomogeneous broadening starts contributing to R_{min} . This effect can phenomenologically be described as the emergence of an effective broadening parameter $\gamma_{eff}(\gamma, \Delta, N)$, which is not a simple combination of γ and Δ , but depends upon N . This parameter is limited from below by γ , when the inhomogeneous broadening is suppressed, and from above by $\tilde{\gamma}$, when the contribution from Δ is largest. Because the minimum value of the reflection is always achieved at a point shifted with respect to ω_d , generally γ_{eff} is always smaller than $\tilde{\gamma}$, and the homogeneous broadening makes a contribution to it comparable to that of the inhomogeneous broadening *despite the fact that $\gamma \ll \Delta$* . At the same time, the position of ω_+ , which determines the width of the spectral interval affected by the defect, depends upon $\tilde{\gamma} \approx \Delta$.

We illustrate this conclusion quantitatively by defining the width of the resonance $W(\gamma, \Delta, N) = \omega_+ - \omega_-$ as a distance between the extrema of the reflection spectrum, and its depth $H(\gamma, \Delta, N) = R_{max} - R_{min}$ as the difference between the values of the reflection at these points. In order to see how these quantities depend upon parameters γ and Δ , we chose several different values of W and H , and plot constant level lines, $W(\gamma, \Delta, N) = W_i, H(\gamma, \Delta, N) = H_i$. These lines represent values of γ and Δ for which W and H remain constant (Fig. 5).

The locus of constant widths is the set of nearly straight lines running almost parallel to the axis representing the homogeneous broadening. Slight deviation from the straight-line behavior is seen only for nonrealistically high values of γ . Such a behavior confirms our assertion that the width of the resonance is determined by an effective parameter, in which γ and Δ enter additively. As we see, this is true even for systems which cannot, strictly speaking, be described by

approximations leading to Eq. (48). Since usually $\gamma \ll \Delta$, the latter makes the largest contribution to this effective parameter, and determines the value of W . The shape of the lines of constant height demonstrates almost equal contributions from γ and Δ , which means that the effect due to the inhomogeneous broadening is significantly reduced as far as this feature of the spectrum is concerned. This is also consistent with an approximate analysis presented in the preceding section of the paper.

The remarkable feature of Fig. 5 is that the lines of the constant width and the constant high cross each other at a rather acute angle and at a single value of γ and Δ for each of the values of W and H . This means that one can extract both γ and Δ from a single reflection spectrum of the MQW structure. This is a rather intriguing opportunity from the experimental point of view, since presently, the only way to independently measure parameters of homogeneous and inhomogeneous broadenings is to use complicated time-resolved techniques.

It is clear, however, that the shape of the lines of constant W and H depends upon the choice of the distribution function used for calculation of the average susceptibility of the defect well. It is important, therefore, to check how the results depend upon the choice of the distribution function of the exciton frequencies. As an extreme example, one can consider the Cauchy distribution

$$f(\omega_0) = \frac{\Delta}{(\omega_0 - \bar{\omega})^2 + \Delta^2}. \quad (50)$$

In this case, all the effects due to the inhomogeneous broadening can be described by a simple renormalization, $\gamma \rightarrow \gamma + \Delta$, and the level lines in Fig. 5 would have the form of parallel lines. This distribution though, hardly has any experimental significance, while the Gaussian function has a certain theoretical justification.³⁷ However, the symmetrical character of the normal distribution is in obvious contradiction with a natural asymmetry of the exciton binding energies, which can be arbitrarily small but are bounded from above. It was suggested in Ref. 25 to take this asymmetry into account by introducing two different variances in the Gaussian distribution: Δ_- for frequencies below some (most probable) frequency ω_c , and Δ_+ for the frequencies above it. Accordingly, the distribution function can be written as

$$f(\omega_0) = \frac{2}{\sqrt{\pi}(\Delta_+ + \Delta_-)} \begin{cases} e^{-(\omega_0 - \omega_c)^2/\Delta_-}, & \omega_0 < \omega_c \\ e^{-(\omega_0 - \omega_c)^2/\Delta_+}, & \omega_0 > \omega_c. \end{cases} \quad (51)$$

It was shown that this choice gives a satisfactory description of time-resolved spectra of MQW's.²⁵ The distribution function, Eq. (51), can be parametrized either by Δ_{\pm} and ω_c or, more traditionally, by the mean value $\bar{\omega}$, the second moment Δ , and the parameter of asymmetry n defined as

$$\bar{\omega} = \omega_c + \frac{\Delta_+ - \Delta_-}{\sqrt{\pi}}, \quad n = \frac{\Delta_+}{\Delta_-},$$

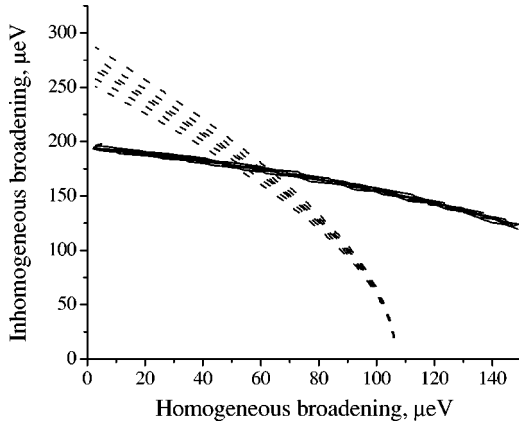


FIG. 6. Intersections for different values of the parameter of asymmetry. Dashed and solids lines are lines of constant height and width, respectively.

$$\Delta^2 = \frac{\Delta_+^3 + \Delta_-^3}{\Delta_+ + \Delta_-} - \frac{2}{\pi} (\Delta_+ - \Delta_-)^2. \quad (52)$$

We use the corrected distribution function, Eq. (51), with the fixed mean frequency and the variance, but different values of the asymmetry parameter, in order to see how sensitive the defect-induced features of the reflection spectrum are to the shape of the distribution function. To this end, we plot the lines of constant height and width for different values of the asymmetry parameter n ($1 \leq n \leq 2$). The results are presented in Fig. 6.

An interesting result revealed by these graphs is that with the change of the asymmetry, the points at which W and H level lines cross move parallel to the axis of γ , while the respective values of Δ remain quite stable. This shift can be explained by noticing that with change of the parameter of asymmetry the total width of the distribution remains the same while the effective value of the inhomogeneous broadening at the point of the reflection changes. This indicates that the value of Δ , which can be obtained by comparing experimental reflection spectra with the theory presented in this paper, is not sensitive to the choice of the distribution function of the exciton frequencies. The value of the parameter of the homogeneous broadening is more sensitive: it varies by approximately 10% when the parameter of the asymmetry changes by a factor of 2. However, the estimate for γ can be improved by studying the temperature dependence of the reflection spectra.

IV. CONCLUSION

In the present paper we studied the reflection spectra of a Bragg multiple quantum well system with a defect: the quantum well at the center of the structure was replaced by a well with a different exciton resonance frequency. In an ideal infinite system such a defect gives rise to a local state with a frequency within the polariton stop band of the host structure, which reveals itself in the form of reflection and transmission resonances.¹⁶ The main focus of this paper was on the effects due to the inhomogeneous broadening, which was

taken into account within the framework of the effective medium approximation.²³ Since in Ref. 23 this approach was introduced on the basis of qualitative arguments only, in the Appendix we presented a rigorous derivation of this approximation and clarified its physical status.

We consider two limiting cases in which defect-induced features of the spectrum can be described analytically. In one case, the length of the system is much larger than the penetration length of the radiation in the infinite periodic structure l_c (deep defect). The modification of the reflection in this case is described by a Lorentz-like minimum, whose depth exponentially decreases with increasing length of the MQW structure, Eq. (42). For long enough structures the presence of the defect in the structure becomes unnoticeable. A much more interesting situation arises in the opposite case of systems much shorter than l_c (shallow defect). In this case, the reflection spectrum exhibits a sharp minimum followed by a maximum. This shape of the spectrum resembles a Fano-like resonance observed in long lossless systems. The position of the maximum of the reflection is close to the exciton frequency in the defect well ω_d , where the short host structure is almost transparent, and therefore the properties of the spectrum near this point are similar to those of an isolated quantum well. At the same time, the minimum is shifted from ω_d by $\Omega_s = -(\omega_d - \omega_h)/N$ and due to this shift an influence of the inhomogeneous broadening Δ on the spectrum near this point is strongly suppressed. When $\Omega_s \gg \Delta$ the value of the reflection at the minimum R_{min} is determined solely by the small homogeneous broadening, and this results in extremely small values of R_{min} , Eq. (49).

The structure of the spectrum with two extrema persists also in the case of an intermediate relation between l_c and N . This situation, however, is more complicated and can only be analyzed numerically. Nevertheless, due to the shift of the minimum reflection frequency one can conclude that the minimum value of the reflection, despite the fact that $\gamma \ll \Delta$, is determined by some effective combination of the homogeneous and inhomogeneous broadenings, while the distance between the extrema is affected mostly by inhomogeneous broadening. This circumstance allowed us suggesting a simple method of extracting both γ and Δ of the defect well from the reflection spectrum of the structure.

ACKNOWLEDGMENTS

We are grateful to Steven Schwarz for reading and commenting on the manuscript. The work is supported by AFOSR Grant No. F49620-02-1-0305 and PSC-CUNY grants.

APPENDIX: THE HORIZONTAL DISORDER AND AVERAGE SUSCEPTIBILITY

A wave incident at a normal z direction on a quantum well can be described by a scalar form of Maxwell's equations for one of the polarizations parallel to the plane of a quantum well:

$$-\nabla^2 E(\rho, z) = \frac{\omega^2}{c^2} [\epsilon_\infty E(\rho, z) + 4\pi P(\rho, z)], \quad (\text{A1})$$

where $P(\rho, z)$ is the polarization due to quantum well excitons. The latter is determined by an expression similar to Eq. (2) with the exciton frequency being a random function of the in-plane coordinate ρ

$$P(\rho, z) = \int dz' \chi(\rho, \omega) \Phi(z) \Phi(z') E(\rho, z'), \quad (\text{A2})$$

where the susceptibility is

$$\chi(\rho, \omega) = \frac{\alpha}{\omega(\rho) - \omega - i\gamma}. \quad (\text{A3})$$

In order to simplify our calculations we make an assumption that $\Phi(z)$ can be approximated by a δ function,

$$\Phi(z) = \Phi_0 \delta(z). \quad (\text{A4})$$

This approximation is sufficient for our particular goals here, but the results obtained will remain valid for more rigorous treatment of the excitonic wave functions as well. After the Fourier transformation with respect to the in-plane coordinates, Eq. (A1) can be presented in the form

$$\begin{aligned} & - \left(\kappa_q^2 + \frac{d^2}{dz^2} \right) E(q, z) \\ & = 2\Phi_0^2 \delta(z) \frac{\omega^2}{c^2} \int d^2 q' \chi(q - q', \omega) E(q', z), \end{aligned} \quad (\text{A5})$$

where $\kappa_q^2 = k^2 - q^2$ and $k = \omega \sqrt{\epsilon_\infty}/c$.

Let us represent the susceptibility as a sum of its average value and a fluctuating part

$$\chi(q, \omega) = \langle \chi(\omega) \rangle + 2\pi \delta(q) + \tilde{\chi}(q, \omega), \quad (\text{A6})$$

where $\langle \chi(\omega) \rangle$ can be written in the form

$$\langle \chi(\omega) \rangle = \int d\omega_0 f(\omega_0) \frac{\alpha}{\omega_0 - \omega - i\gamma}. \quad (\text{A7})$$

The structure of the Maxwell equation dictates that the reflection and transmission coefficients have a δ -functional singularity in the specular direction,³⁸ $q=0$. Thus the wave

at the left-hand side of the quantum well E_- and at the right-hand side E_+ can be presented in the following form

$$\begin{aligned} E_-(q, z) &= 2\pi E_0 (e^{ikz} + r_0 e^{-ikz}) \delta(q) + E_0 r(q) e^{-i\kappa_q z}, \\ E_+(q, z) &= 2\pi E_0 t_0 e^{ikz} \delta(q) + E_0 t(q) e^{i\kappa_q z}. \end{aligned} \quad (\text{A8})$$

After the substitution of Eqs. (A7) and (A8) into Eq. (A5) we obtain an integral equation for the reflection and transmission coefficients $r(q)$ and $t(q)$. Assuming that the random process representing the fluctuating part of χ does not contain constant or almost periodic realizations, we conclude that $\tilde{\chi}(q, \omega)$ does not have δ -like singularities in almost all realizations. In this case, the terms proportional to $\delta(q)$ in Eq. (A5), this equation must cancel each other independently of other terms. This leads to a system of equations for r_0 and t_0 with the solutions

$$r_0 = \frac{i\eta}{1 - i\eta}, \quad t_0 = \frac{1}{1 - i\eta}, \quad (\text{A9})$$

where

$$\eta = \int d\omega_0 f(\omega_0) \frac{\Gamma_0}{\omega_0 - \omega - i\gamma}, \quad (\text{A10})$$

with Γ_0 defined by Eq. (8) with the envelope wave function given by Eq. (A4).

These expressions coincide with those of the effective medium approach (EMA), Eq (6), with an accuracy up to the phase factor, which does not appear here because of the δ -functional approximation for the excitonic wave function.

This derivation shows that the scattered wave consists of the random and deterministic (singular) components. The latter determines the energy flux through a detector for measurements at the specular direction with a small angular aperture and, thus, is directly measurable. The important feature of the deterministic component is that it obeys the Maxwell equations with averaged susceptibility. In other words, EMA describes only the specular component of the scattered field, but it describes this component exactly in the limit of the infinite sample. From experimental point of view the accuracy of EMA is limited by the finite angular resolution of the detector: the better the resolution the more accurately EMA describes the experimental results.

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