CHAPTER

5 CIRCULAR MOTION

1. ***REASONING*** The speed of the plane is given by Equation 5.1: $v = 2\pi r/T$, where T is the period or the time required for the plane to complete one revolution.

SOLUTION Solving Equation 5.1 for T we have

$$T = \frac{2\pi r}{v} = \frac{2\pi(2850\text{ m})}{110\text{ m/s}} = \boxed{160\text{ s}}$$

5. ***REASONING*** The magnitude a_c of the car's centripetal acceleration is given by Equation 5.2 as $a_c = v^2/r$, where v is the speed of the car and r is the radius of the track. The radius is $r = 2.6 \times 10^3$ m. The speed can be obtained from Equation 5.1 as the circumference ($2\pi r$) of the track divided by the period T of the motion. The period is the time for the car to go once around the track ($T = 360$ s).

SOLUTION Since $a_c = v^2/r$ and $v = (2\pi r)/T$, the magnitude of the car's centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(2.6\times 10^3\text{ m}\right)}{(360\text{ s})^2} = \boxed{0.79\text{ m/s}^2}$$

9. ***REASONING AND SOLUTION*** Since the magnitude of the centripetal acceleration is given by Equation 5.2, $a_c = v^2/r$, we can solve for r and find that

$$r = \frac{v^2}{a_C} = \frac{(98.8\text{ m/s})^2}{3.00(9.80\text{ m/s}^2)} = \boxed{332\text{ m}}$$

15. ***REASONING AND SOLUTION*** The magnitude of the centripetal force on the ball is given by Equation 5.3: $F_C = mv^2/r$. Solving for v, we have

$$v = \sqrt{\frac{F_C r}{m}} = \sqrt{\frac{(0.028\text{ N})(0.25\text{ m})}{0.015\text{ kg}}} = \boxed{0.68\text{ m/s}}$$

33. ***REASONING*** Equation 5.5 gives the orbital speed for a satellite in a circular orbit around the earth. It can be modified to determine the orbital speed around any planet **P** by replacing the mass of the earth by M_E by the mass of the planet M_P: $v = \sqrt{GM_P/r}$.

SOLUTION The ratio of the orbital speeds is, therefore,

$$\frac{v_2}{v_1} = \frac{\sqrt{GM_P/r_2}}{\sqrt{GM_P/r_1}} = \sqrt{\frac{r_1}{r_2}}$$

Solving for v_2 gives

$$v_2 = v_1\sqrt{\frac{r_1}{r_2}} = (1.70\times 10^4\text{ m/s})\sqrt{\frac{5.25\times 10^6\text{ m}}{8.60\times 10^6\text{ m}}} = \boxed{1.33\times 10^4\text{ m/s}}$$

37. ***REASONING*** Equation 5.2 for the centripetal acceleration applies to both the plane and the satellite, and the centripetal acceleration is the same for each. Thus, we have

$$a_c = \frac{v_{\text{plane}}^2}{r_{\text{plane}}} = \frac{v_{\text{satellite}}^2}{r_{\text{satellite}}} \quad \text{or} \quad v_{\text{plane}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right) v_{\text{satellite}}$$

The speed of the satellite can be obtained directly from Equation 5.5.

SOLUTION Using Equation 5.5, we can express the speed of the satellite as

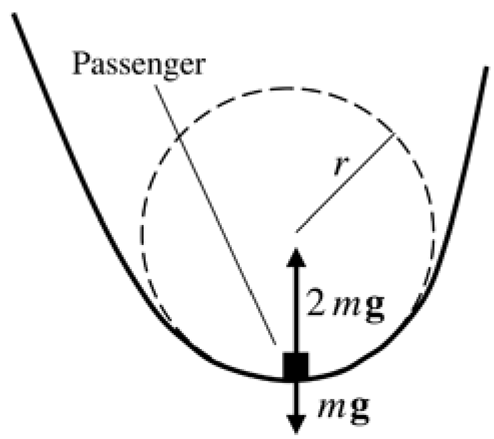
$$v_{\text{satellite}} = \sqrt{\frac{Gm_E}{r_{\text{satellite}}}}$$

Substituting this expression into the expression obtained in the reasoning for the speed of the plane gives

$$v_{\text{plane}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right) v_{\text{satellite}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right) \sqrt{\frac{Gm_E}{r_{\text{satellite}}}} = \frac{\sqrt{r_{\text{plane}} G m_E}}{r_{\text{satellite}}}$$

$$v_{\text{plane}} = \frac{\sqrt{(15\text{ m})\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{kg}\right)}}{6.7\times10^6\text{ m}} = \boxed{12\text{ m/s}}$$

41. ***REASONING*** According to Equation 5.3, the magnitude F_c of the centripetal force that acts on each passenger is $F_c = mv^2/r$, where m and v are the mass and speed of a passenger and r is the radius of the turn. From this relation we see that the speed is given by $v = \sqrt{F_c r/m}$. The centripetal force is the net force required to keep each passenger moving on the circular path and points toward the center of the circle. With the aid of a free-body diagram, we will evaluate the net force and, hence, determine the speed.



SOLUTION The free-body diagram shows a passenger at the bottom of the circular dip. There are two forces acting: her downward-acting weight $m\mathbf{g}$ and the upward-acting force $2m\mathbf{g}$ that the seat exerts on her. The net force is $+2mg - mg = +mg$, where we have taken "up" as the positive direction. Thus, $F_c = mg$. The speed of the passenger can be found by using this result in the equation above.

Substituting $F_c = mg$ into the relation $v = \sqrt{F_c r/m}$ yields

$$v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{(mg)r}{m}} = \sqrt{g r} = \sqrt{\left(9.80\text{ m/s}^2\right)(20.0\text{ m})} = \boxed{14.0\text{ m/s}}$$

43. ***REASONING*** The centripetal force is the name given to the net force pointing toward the center of the circular path. At point 3 at the top the net force pointing toward the center of the circle consists of the normal force and the weight, both pointing toward the center. At point 1 at the bottom the net force consists of the normal force pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as $F_C = mv^2/r$.

SOLUTION At point 3 we have

$$F_c = F_N + mg = \frac{mv_3^2}{r}$$

At point 1 we have

$$F_c = F_N - mg = \frac{mv_1^2}{r}$$

Subtracting the second equation from the first gives

$$2mg = \frac{mv_3^2}{r} - \frac{mv_1^2}{r}$$

Rearranging gives

$$v_3^2 = 2gr + v_1^2$$

Thus, we find that

$$v_3 = \sqrt{2(9.80\text{ m/s}^2)(3.0\text{ m}) + (15\text{ m/s})^2} = \boxed{17\text{ m/s}}$$

49. ***REASONING*** In Example 3, it was shown that the magnitudes of the centripetal acceleration for the two cases are

$$[\text{Radius} = 33\text{ m}]\quad a_C = 35\text{ m/s}^2$$

$$[\text{Radius} = 24\text{ m}]\quad a_C = 48\text{ m/s}^2$$

According to Newton's second law, the centripetal force is $F_C = ma_C$ (see Equation 5.3).

SOLUTION

a. Therefore, when the sled undergoes the turn of radius 33 m,

$$F_C = ma_C = (350\text{ kg})(35\text{ m/s}^2) = \boxed{1.2 \times 10^4\text{ N}}$$

b. Similarly, when the radius of the turn is 24 m,

$$F_C = ma_C = (350\text{ kg})(48\text{ m/s}^2) = \boxed{1.7 \times 10^4\text{ N}}$$

55. ***REASONING*** As the motorcycle passes over the top of the hill, it will experience a centripetal force, the magnitude of which is given by Equation 5.3: $F_C = mv^2/r$. The centripetal force is provided by the net force on the cycle + driver system. At that instant, the net force on the system is composed of the normal force, which points upward, and the weight, which points downward. Taking the direction toward the center of the circle (downward) as the positive direction, we have $F_C = mg - F_N$. This expression can be solved for F_N , the normal force.

SOLUTION

a. The magnitude of the centripetal force is

$$F_C = \frac{mv^2}{r} = \frac{(342\text{ kg})(25.0\text{ m/s})^2}{126\text{ m}} = \boxed{1.70 \times 10^3\text{ N}}$$

b. The magnitude of the normal force is

$$F_N = mg - F_C = (342\text{ kg})(9.80\text{ m/s}^2) - 1.70 \times 10^3\text{ N} = \boxed{1.66 \times 10^3\ N}$$

57. ***REASONING AND SOLUTION*** The centripetal acceleration for any point on the blade a distance r from center of the circle, according to Equation 5.2, is $a_c = v^2/r$. From Equation 5.1, we know that $v = 2\pi r/T$ where T is the period of the motion. Combining these two equations, we obtain

$$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

a. Since the turbine blades rotate at 617 rev/s, all points on the blades rotate with a period of $T = (1/617)s = 1.62 \times 10^{-3}$ s. Therefore, for a point with r = 0.020 m, the magnitude of the centripetal acceleration is

$$a_c = \frac{4\pi^2(0.020\text{ m})}{(1.62 \times 10^{+3}\text{ s})^2} = \boxed{3.0 \times 10^5\text{ m/s}^2}$$

b. Expressed as a multiple of g, this centripetal acceleration is

$$a_c = \left(3.0 \times 10^5\text{ m/s}^2\right)\left(\frac{1.00g}{9.80\text{ m/s}^2}\right) = \boxed{3.1 \times 10^4\ g}$$

59. ***REASONING*** Let be the initial speed of the ball as it begins its projectile motion. Then, the centripetal force is given by Equation 5.3: $F_c = mv_0^2/r$. We are given the values for m and r; however, we must determine the value of from the details of the projectile motion after the ball is released.

In the absence of air resistance, the x component of the projectile motion has zero acceleration, while the y component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the ball is given by Equation 3.5a (with a_x = m/s^2):

$$x = v_{0x}t = (v_0 \cos\theta)t$$

with t equal to the flight time of the ball while it exhibits projectile motion. The time t can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_y t$$

After a time t, $v_y = -v_0y$. Assuming that up and to the right are the positive directions, we have

$$t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin\theta}{a_y}$$

and

$$x = (v_0 \cos\theta)\left(\frac{-2v_0 \sin\theta}{a_y}\right)$$

Using the fact that 2 sinθ cosθ = sin 2θ, we have

$$x = -\frac{2v_0^2 \cos\theta \sin\theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y} \qquad (1)$$

Equation 1 (with upward and to the right chosen as the positive directions) can be used to determine the speed $v_0$with which the ball begins its projectile motion. Then Equation 5.3 can be used to find the centripetal force.

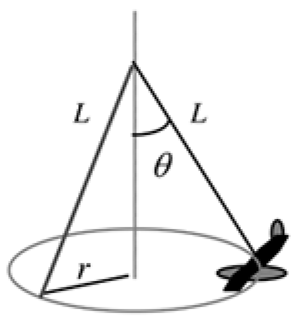
SOLUTION Solving equation 1 for v_0 , we have

$$v_0 = \sqrt{\frac{-x\, a_y}{\sin 2\theta}} = \sqrt{\frac{-(86.75\text{ m})(-9.80\text{ m/s}^2)}{\sin 2(41°)}} = 29.3\text{ m/s}$$

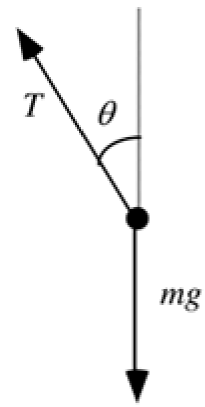
Then, from Equation 5.3,

$$F_C = \frac{mv_0^2}{r} = \frac{(7.3\text{ kg})(29.3\text{ m/s})^2}{1.8\text{ m}} = \boxed{3500\text{ N}}$$

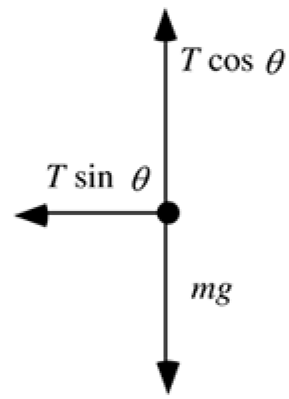
61. ***REASONING*** If the effects of gravity are not ignored in Example 5, the plane will make an angle with the vertical as shown in figure **A** below. The figure **B** shows the forces that act on the plane, and figure **C** shows the horizontal and vertical components of these forces.



A



B



C

From figure **C** we see that the resultant force in the horizontal direction is the horizontal component of the tension in the guideline and provides the centripetal force. Therefore,

$$T \sin\theta = \frac{mv^2}{r}$$

From figure **A**, the radius r is related to the length L of the guideline by $r = L\sin\theta$; therefore,

$$T \sin\theta = \frac{mv^2}{L\sin\theta} \qquad (1)$$

The resultant force in the vertical direction is zero: $T\cos\theta - mg = 0$, so that

$$T\cos\theta = mg \qquad (2)$$

From equation 2 we have

$$T = \frac{mg}{\cos\theta} \qquad (3)$$

Equation 3 contains two unknown, T and θ. First we will solve Equations 1 and 3 simultaneously to determine the value(s) of the angle θ. Once θ is known, we can calculate the tension using equation 3.

SOLUTION Substituting equation 3 into equation 1:

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{mv^2}{L\sin\theta}$$

Thus,

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{gL} \qquad (4)$$

Using the fact that cos2 θ + sin2 θ = 1, equation 4 can be written

$$\frac{1 \pm \cos^2\theta}{\cos\theta} = \frac{v^2}{gL}$$

or

$$\frac{1}{\cos\theta} - \cos\theta = \frac{v^2}{gL}$$

This can be put in the form of an equation that is quadratic in cos θ. Multiplying both sides by cos θ and rearranging yields:

$$\cos^2\theta + \frac{v^2}{gL}\cos\theta \pm 1 = 0 \qquad (5)$$

Equation 5 is of the form

$$ax^2 + bx + c = 0 \qquad (6)$$

with $x = \cos\theta$, $a = 1$, $b = v_2(gL)$, and $c = -1$. The solution to equation 6 is found from the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When $v = 19.0$ m/s, $b = 2.17$. The positive root from the quadratic formula gives $x = \cos\theta = 0.391$. Substitution into equation 3 yields

$$T = \frac{mg}{\cos\theta} = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)}{0.391} = \boxed{23\text{ N}}$$

When $v = 38.0$ m/s, $b = 8.67$. The positive root from the quadratic formula gives $x = \cos\theta = 0.114$. Substitution into equation 3 yields

$$T = \frac{mg}{\cos\theta} = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)}{0.114} = \boxed{77\text{ N}}$$